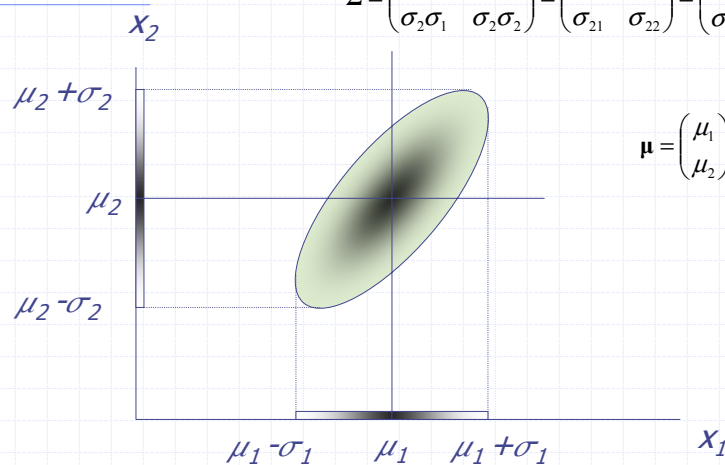


Using R for Data Analysis: Populations and linear regression

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Parameters of the multidimensional Gaussian

$$\Sigma = \begin{pmatrix} \sigma_1\sigma_1 & \sigma_1\sigma_2 \\ \sigma_2\sigma_1 & \sigma_2\sigma_2 \end{pmatrix} = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{pmatrix}$$



$$p(\mathbf{x}) = \frac{1}{(2\pi)^{d/2} |\Sigma|^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu})' \Sigma^{-1} (\mathbf{x} - \boldsymbol{\mu})\right] \text{ abbreviated : } N(\boldsymbol{\mu}, \Sigma)$$

Two-dimensional Gaussian

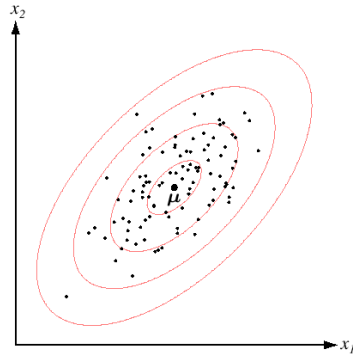


FIGURE 2.9. Samples drawn from a two-dimensional Gaussian lie in a cloud centered on the mean μ . The ellipses show lines of equal probability density of the Gaussian. From: Richard O. Duda, Peter E. Hart, and David G. Stork, *Pattern Classification*. Copyright © 2001 by John Wiley & Sons, Inc.

Independent and dependent distributions

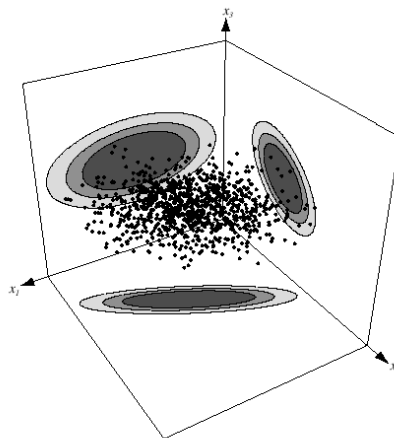


FIGURE 2.23. A three-dimensional distribution which obeys $p(x_1, x_3) = p(x_1)p(x_3)$; thus here x_1 and x_3 are statistically independent but the other feature pairs are not. From: Richard O. Duda, Peter E. Hart, and David G. Stork, *Pattern Classification*. Copyright © 2001 by John Wiley & Sons, Inc.

Linear Transformations

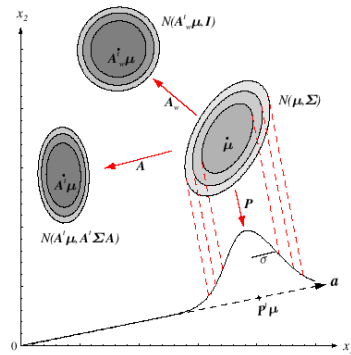


FIGURE 2.8. The action of a linear transformation on the feature space will convert an arbitrary normal distribution into another normal distribution. One transformation, A , takes the source distribution into distribution $N(A\mu, A\Sigma A)$. Another linear transformation—a projection P onto a line defined by vector a —leads to $N(\mu, \sigma^2)$ measured along that line. While the transforms yield distributions in a different space, we show them superimposed on the original x_1, x_2 -space. A whitening transform, A_w , leads to a circularly symmetric Gaussian, here shown displaced. From: Richard O. Duda, Peter E. Hart, and David G. Stork, *Pattern Classification*. Copyright © 2001 by John Wiley & Sons, Inc.